Constraining the ρ meson wavefunction*

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Abstract

Diffractive ρ meson production has been identified as one of the important processes where saturation can be probed at a future Electron-Ion Collider (EIC). A source of uncertainty in making predictions for this process lies within the assumed form of the meson light-cone wavefunction. We report here the results of reference [1] where a Regge-inspired dipole model was used to extract this wavefunction as well as the corresponding leading twist-2 Distribution Amplitude from the current accurate HERA data. In addition, we shall check the robustness of the conclusions in reference [1] by using alternative Colour Glass Condensate dipole models.

1. Introduction

In the dipole model [2, 3], the imaginary part of the amplitude for diffractive ρ production is written as [4]

$$\Im\mathrm{m}\mathcal{A}_\lambda(s,t;Q^2) = \sum_{h,\bar{h}} \int \mathrm{d}^2\mathbf{r}\mathrm{d}z \Psi^{\gamma^*,\lambda}_{h,\bar{h}}(r,z;Q^2)\Psi^{\rho,\lambda}_{h,\bar{h}}(r,z)^* e^{-iz\mathbf{r}.\mathbf{\Delta}}\mathcal{N}(x,\mathbf{r},\mathbf{\Delta}) \tag{1}$$

where $t=-|\mathbf{\Delta}|^2$. In a standard notation [1, 4, 5], $\Psi^{\gamma^*,\lambda}_{h,\bar{h}}$ and $\Psi^{\rho,\lambda}_{h,\bar{h}}$ are the light-cone wavefunctions of the photon and the ρ meson respectively while $\mathcal{N}(x,\mathbf{r},\mathbf{\Delta})$ is the imaginary part of the dipole-proton elastic scattering amplitude. The energy dependence of the latter is via the dimensionless variable x taken here to be

$$x = (Q^2+4m_f^2)/(Q^2+s) \tag{2}$$

where m_f is a phenomenological light quark mass.[1] Setting $t=0$ in equation (1), we obtain the forward amplitude used in reference [1], i.e

$$\Im\mathrm{m}\,\mathcal{A}_\lambda(s,t;Q^2)\big|_{t=0} = s\sum_{h,\bar{h}}\int \mathrm{d}^2\mathbf{r}\;\mathrm{d}z\;\Psi^{\gamma,\lambda}_{h,\bar{h}}(r,z;Q^2)\hat{\sigma}(x,r)\Psi^{\rho,\lambda}_{h,\bar{h}}(r,z)^* \tag{3}$$

where we have used the optical theorem to introduce the dipole cross-section

$$\hat{\sigma}(x,r) = \frac{\mathcal{N}(x,r,\mathbf{0})}{s}\;. \tag{4}$$

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[1] Here we shall take $m_f = 0.14$ GeV i.e the value used when extracting the dipole cross-section from F_2 data.

Note that since the momentum transfer $\mathbf{\Delta}$ is Fourier conjugate to the impact parameter $\mathbf{b}$, the dipole cross-section at a given energy is simply the b-integrated dipole-proton scattering amplitude, i.e

$$\hat{\sigma}(x,r) = \frac{1}{s}\int \mathrm{d}^2\mathbf{b}\mathcal{N}(x,r,\mathbf{b})\ . \tag{5}$$

This dipole cross-section can be extracted from the F_2 data since

$$F_2(x,Q^2) \propto \int \mathrm{d}^2\mathbf{r}\mathrm{d}z|\Psi_{\gamma^*}(r,z;Q^2)|^2\hat{\sigma}(x,r) \tag{6}$$

and the photon's light-cone wavefunctions are known in QED, at least for large Q^2. The F_2-constrained dipole cross-section can then be used to predict the imaginary part of the forward amplitude for diffractive ρ production and thus the forward differential cross-section,

$$\left.\frac{d\sigma_\lambda}{dt}\right|_{t=0} = \frac{1}{16\pi}(\Im\mathrm{m}\mathcal{A}_\lambda(s,0))^2\ (1+\beta_\lambda^2)\ , \tag{7}$$

where β_λ is the ratio of real to imaginary parts of the amplitude and is computed as in reference [1]. The t-dependence can be assumed to be the exponential dependence as suggested by experiment [6]:

$$\frac{d\sigma_\lambda}{dt} = \left.\frac{d\sigma_\lambda}{dt}\right|_{t=0} \times \exp(-B|t|) \tag{8}$$

where

$$B = N\left(14.0\left(\frac{1\ \mathrm{GeV}^2}{Q^2+M_\rho^2}\right)^{0.2}+1\right) \tag{9}$$

with $N = 0.55\ \mathrm{GeV}^{-2}$. After integrating over t, we can compute the total cross-section $\sigma = \sigma_L + \epsilon\sigma_T$ which is measured at HERA.[2]

Presently several dipole models [7, 8, 9, 10, 11] are able to fit the current HERA F_2 data and there is evidence that the data prefer those incorporating some form of saturation [12]. We can use the F_2-constrained dipole cross-section in order to extract the ρ light-cone wavefunction using the current precise HERA data [6, 13]. This has recently been done in reference [1] using the Regge-inspired FSSat dipole model [7] and we shall report the results of this work here. In addition, we repeat the analysis using two alternative models [9, 8, 11] both based on the original Colour Glass Condensate (CGC) model [14]. They differ from the original CGC model by including the contribution of charm quarks when fitting to the F_2 data. Furthermore in one of them [9, 8], the anomalous dimension γ_s is treated as an additional free parameter instead of being fixed to its LO BFKL value of 0.63. We shall refer to these models as CGC[0.74] and CGC[0.63] models where the number in the square brackets stands for the fitted and fixed value of the anomalous dimension respectively. For both models, we use the set of fitted parameters given in reference [8]. All three models, i.e FSSat, CGC[0.63] and CGC[0.74] account for saturation although in a b- (or equivalently t-) independent way. Indeed, at a given energy, the dipole cross-section is equal to the forward dipole-proton amplitude given by equation (4) or to the b-integrated dipole proton amplitude given by equation (5). Finally we note that all three dipole models we consider here give a good description of the diffractive structure function, i.e $F_2^{D(3)}$ data [15, 16].

[2]To compare with the HERA data, we take $\epsilon = 0.98$.

2. Fitting the HERA data

Previous work [5, 4, 8] has shown that a reasonable assumption for the scalar part of the light-cone wavefunction for the ρ is of the form

$$\begin{aligned}\phi_\lambda^{\mathrm{BG}}(r,z) &= \mathcal{N}_\lambda\, 4[z(1-z)]^{b_\lambda}\sqrt{2\pi R_\lambda^2}\, \exp\left(\frac{m_f^2 R_\lambda^2}{2}\right)\exp\left(-\frac{m_f^2 R_\lambda^2}{8[z(1-z)]^{b_\lambda}}\right) \\ &\quad \times \exp\left(-\frac{2[z(1-z)]^{b_\lambda} r^2}{R_\lambda^2}\right)\end{aligned} \tag{10}$$

and is referred to as the 'Boosted Gaussian' (BG). This wavefunction is a simplified version of that proposed originally by Nemchik, Nikolaev, Predazzi and Zakharov [17]. In the original BG wavefunction, $b_\lambda = 1$ while the parameters R_λ and $\mathcal{N}_\lambda$ are fixed by the leptonic decay width constraint and the wavefunction normalization conditions [1].

However, when the BG wavefunction is used in conjunction with either the FSSat model or any of the CGC models, none of them is able to give a good quantitative agreement with the current HERA ρ-production data. This is illustrated by the large χ^2 values in table 1. As shown in table 2, this situation is considerably improved by fitting R_λ and b_λ to the leptonic decay width and HERA data.[3]

Boosted Gaussian predictions

Dipole model	χ^2/data point
FSSat	$310/75$
CGC[0.74]	$262/75$
CGC[0.63]	$401/75$

Table 1: Predictions of the χ^2/data point using the BG wavefunction.

BG fits

Model	χ^2/d.o.f
FSSat [1]	$82/72$
CGC[0.74]	$64/72$
CGC[0.63]	$83/72$

Table 2: χ^2/d.o.f obtained when fitting R_λ and b_λ to the leptonic decay width and HERA data.

For the FSSat and CGC[0.63] models, we can further improve the quality of fit by allowing for additional end-point enhancement in the transverse wavefunction, i.e. using a scalar wavefunction of the form

$$\phi_T(r,z) = \phi_T^{\mathrm{BG}}(r,z) \times [1 + c_T\xi^2 + d_T\xi^4] \tag{11}$$

where $\xi = 2z - 1$. The results are shown in table 3.

[3] We fit to the same data set and with the same cuts as in reference [1].

Improved fits

Model	χ^2/d.o.f
FSSat [1]	$68/70$
CGC[0.63]	$67/70$

Table 3: χ^2/d.o.f obtained when fitting b_λ, R_λ c_T, d_T the leptonic decay width and HERA data.

Best fit parameters

	R_L^2	R_T^2	b_L	b_T	c_T	d_T
FSSat [1]	26.76	27.52	0.5665	0.7468	0.3317	1.310
CGC[0.63]	27.31	31.92	0.5522	0.7289	1.6927	2.1457
CGC[0.74]	26.67	21.30	0.5697	0.7929	0	0

Table 4: Best fit parameters for each dipole model.

The best fits obtained with each dipole model are compared to the HERA data in figure 1, 2 and 3. The corresponding fitted parameters are given in table 4. Note that we achieve a lower $\chi^2/\text{d.o.f} = 0.89$ with CGC[0.74] than with CGC[0.63] and FSSat for which we obtain $\chi^2/\text{d.o.f} = 0.96$ and $\chi^2/\text{d.o.f} = 0.97$ respectively. Compared to the FSSat and CGC[0.63] fits, note that no additional enhancement in the transverse wavefunction is required in the CGC[0.74] fit. Nevertheless the extracted wavefunction still exhibits enhancement compared to the old BG wavefunction. The extracted light-cone wavefunctions are shown in figure 4.

3. Distribution Amplitudes

The leading twist-2 Distribution Amplitude (DA) is given by [1]

$$\varphi(z,\mu) \sim \left(1 - \mathrm{e}^{-\mu^2/\Delta(z)^2}\right) \mathrm{e}^{-m_f^2/\Delta(z)^2} [z(1-z)]^{b_L} \;, \tag{12}$$

where $\Delta(z)^2 = 8[z(1-z)]^{b_L}/R_L^2$. This leading twist DA is only sensitive to the longitudinal wavefunction and, as illustrated in figure 5, we expect little variation in the predictions using the different dipole models. To compare with existing theoretical predictions for the DA, we compute moments, i.e.

$$\langle \xi^n \rangle_\mu = \int_0^1 \mathrm{d}z \; \xi^n \varphi(z,\mu) \; . \tag{13}$$

where by convention [1]

$$\int_0^1 \mathrm{d}z \; \varphi(z,\mu) = 1 \; . \tag{14}$$

In reference [1], we noted that our DA is very slowly varying with μ for $\mu > 1$ GeV, i.e our parameterization neglects the perturbatively known μ-dependence of the DA. This statement remains true if we use the CGC[0.63] or CGC[0.74] instead of the FSSat model.

Our results are compared with the existing predictions in table 5. The moments obtained with our best fit, i.e with the CGC[0.74] model, are very similar to those obtained with FSSat model or

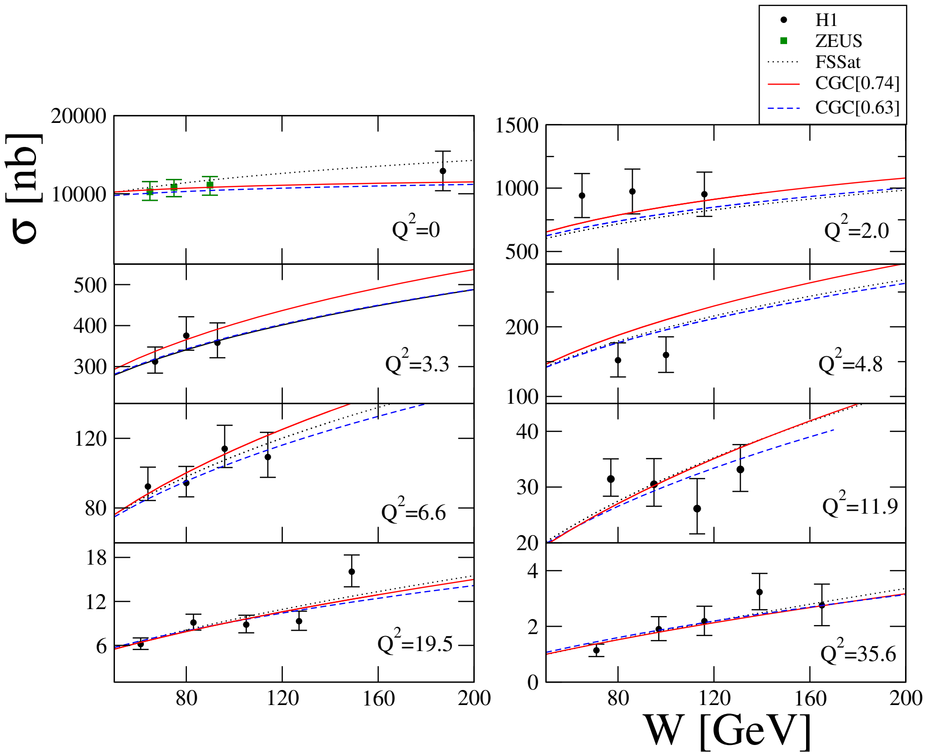

Fig. 1: Best fits to the HERA total cross-section data. CGC[0.74]: solid; FSSat: dotted; CGC[0.63]: dashed.

the CGC[0.63]. In all cases, the results are in very good agreement with expectations based on QCD sum rules and the lattice.

Finally, in figure 5 we compare our DAs with that predicted by Ball and Braun [20], at a scale $\mu = 1$ GeV. The agreement is reasonable given that in reference [20], the expansion in Gegenbauer polynomials is truncated at low order, which is presumably responsible for the local minimum at $z = 1/2$. Certainly all 4 distributions distributions are broader than the asymptotic prediction $\sim 6z(1-z)$.

4. Conclusions

We have used the current HERA data on diffractive ρ production to extract information on the ρ light-cone wavefunction. We find that the corresponding leading twist-2 DA is broader than the asymptotic shape and agrees very well with the expectations of QCD sum rules and the lattice. We also find that the data prefer a transverse wavefunction with end-point enhancement although the degree of such an enhancement is model-dependent.

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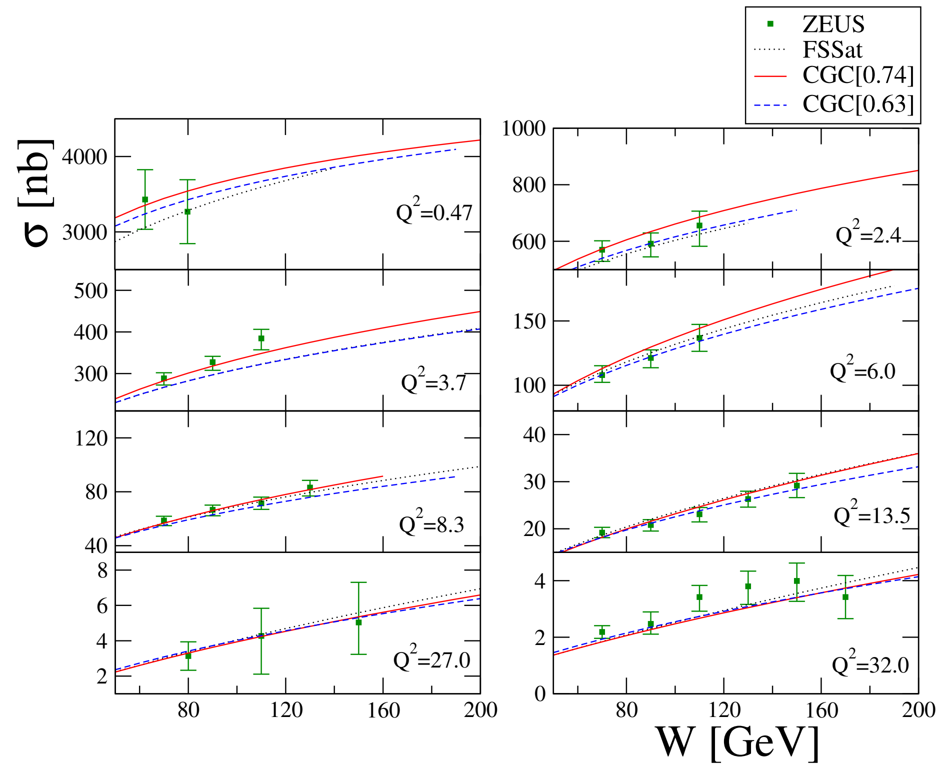

Fig. 2: Best fits to the ZEUS total cross-section data. CGC[0.74]: solid; FSSat: dotted; CGC[0.63]: dashed.

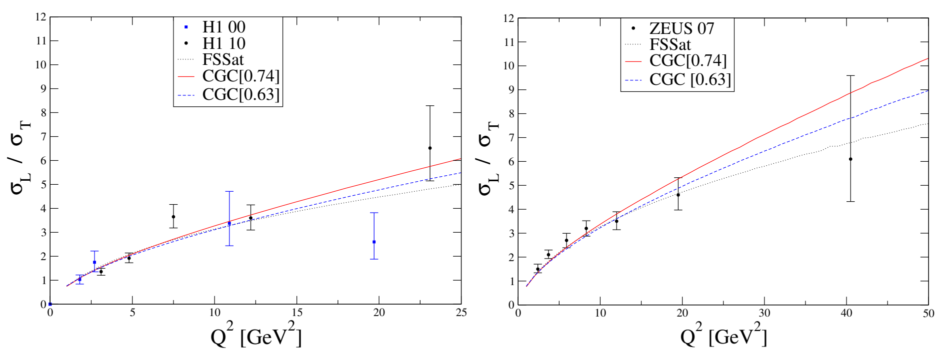

Fig. 3: Best fits to the σ_L/σ_T data. The H1 data are at $W = 75$ GeV while the ZEUS data are at $W = 90$ GeV. CGC[0.74]: solid; FSSat: dotted; CGC[0.63]: dashed.

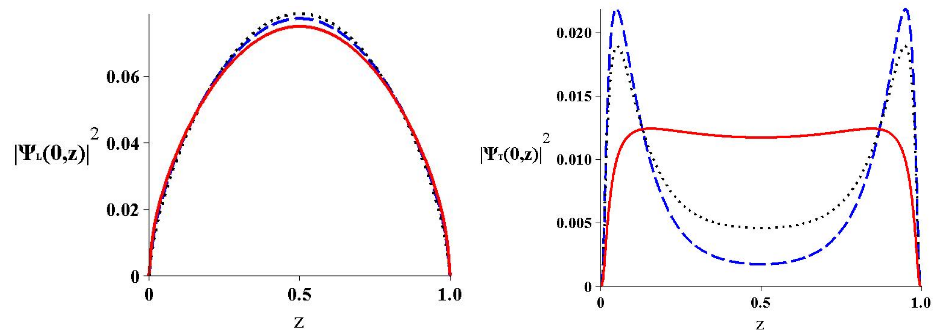

Fig. 4: The longitudinal (left) and transverse (right) light-cone wavefunctions squared at $r = 0$. CGC[0.74]: solid; FSSat: dotted; CGC[0.63]: dashed.

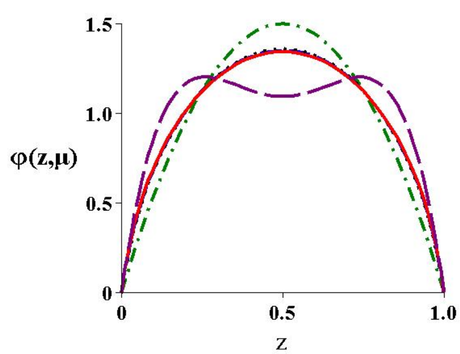

Fig. 5: The extracted leading twist-2 DAs at $\mu = 1$ GeV (CGC[0.74]: solid; FSSat: dotted; CGC[0.63]: dashed) compared to the DA of reference [20] also at 1 GeV (long-dashed) and the asymptotic DA (dot-dashed).

Moments of the leading twist DA at the scale μ

Reference	Approach	Scale μ	$\langle\xi^2\rangle_\mu$	$\langle\xi^4\rangle_\mu$	$\langle\xi^6\rangle_\mu$	$\langle\xi^8\rangle_\mu$	$\langle\xi^{10}\rangle_\mu$
(This paper)	CGC[0.74] fit	~ 1 GeV	0.227	0.105	0.062	0.041	0.029
(This paper)	CGC[0.63] fit	~ 1 GeV	0.229	0.107	0.063	0.042	0.030
[1]	FSSat fit	~ 1 GeV	0.227	0.105	0.062	0.041	0.029
(This paper)	Old BG prediction	~ 1 GeV	0.181	0.071	0.036	0.021	0.014
[18]	GenSR	1 GeV	$0.227(7)$	$0.095(5)$	$0.051(4)$	$0.030(2)$	$0.020(5)$
[19]	SR	1 GeV	0.26	0.15			
[20]	SR	1 GeV	$0.26(4)$				
[21]	SR	1 GeV	0.254				
[22]	SR	1 GeV	$0.23\pm^{0.03}_{0.02}$	$0.11\pm^{0.03}_{0.02}$			
[23]	Lattice	2 GeV	$0.24(4)$				
	$6z(1-z)$	∞	0.2	0.086	0.048	0.030	0.021

Table 5: Our extracted values for $\langle\xi^n\rangle_\mu$, compared to predictions based on the QCD sum rules (SR), Generalised QCD Sum Rules (GenSR) or lattice QCD.

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